Vibration Control of Beams using Absorber

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ABSTRACT

The present work focuses on the analysis of the effectiveness of dynamic vibration absorber applied to cantilever beams excited by moving loads. In this project we have considered a cantilever beam for the vibration analysis. Analytical solution for a beam with varying geometry are complex and hence numerical methods like FEM is used in modal analysis of the beam due to first few modes since higher modes do not contribute to the total response. Equations of motions have been developed for cantilever beam with and without absorber and solved numerically by developing a code in MATLAB. The response results were validated using ANSYS work bench. The results obtained by MATLAB were also validated with analytical solutions.

KEY WORDS: cantilever beam, dynamic vibration absorber, vibration control, FEM.

1. INTRODUCTION

The motivation of this work stems from the possibility of using vibration absorbers to damp out the undesirable responses which are an unmitigated evil. The task of reducing the undesirable effects of resonant disturbances has been tackled employing a variety of approaches ranging from the introduction of active dynamic absorber as well as passive dynamic absorbers. For example, a number of works have shown both theoretically and experimentally the problem of controlling the vibration by using active dynamic absorbers. Work on DVAs was done rigorously from the development of helicopter rotor blades since 1963, and recently, for the defense mechanism against earthquakes. A force generator developed by Rockwell (1965) also acting as the absorber mass can be mounted on the beam. A sensor mounted on the other side of the beam detects the motion of the beam and sends a feedback signal to the generator, which in turn reacts against the motion of the vibration.

Numerous applications involving active control of Dynamic Absorbers have dealt with actuating the absorber mass directly. This study is different in the way that the “active” component is implemented. This method of control has the potential to be less energy consuming, as the power required to adjust the spring variable is expected to be less than the power required to actuate the mass directly. This method is best described as an adaptive-passive approach to vibration control. Adaptive Helmholtz resonators, described by are an example of where adaptive-passive methods have been used for narrowband applications. In the area of broadband applications, explains the use of adaptive-passive methods to vary the stiffness and damping of an engine mount. In the area of structural control.

The results from this study indicated that the use of adaptive-passive vibration absorbers is feasible in the control of vibration. Self-adapting absorbers have been previously used in the vibration control of unbalanced rotating shafts. These come in the form of centrifugal absorbers, which can vary their pendulum angle in accordance with the angular speed. This results in the stiffness of the absorber increasing at a rate square to the angular speed, which means the Eigen frequency increases directly with the speed of the rotating. So in the present work of controlling the vibration we have been done using the passive dynamic absorber. The absorber has been modelled as a spring mass system. In terms of the installations and maintenance of control devices passive methods are cost effective are thus widely used today.

Theoritical Analysis of Transverse Vibration of Fixed Free Beam: Consider slender beam subjected to transverse vibration. Here, M(x, t) is the bending moment, V(x, t) is the shear force, and $f(x,t)$ is the external force per unit length of the beam. Since the inertia force acting on the element of the beam is ρA(x) $\frac{d^2w}{dx^2}(x,t)$ balancing the forces in z direction gives:

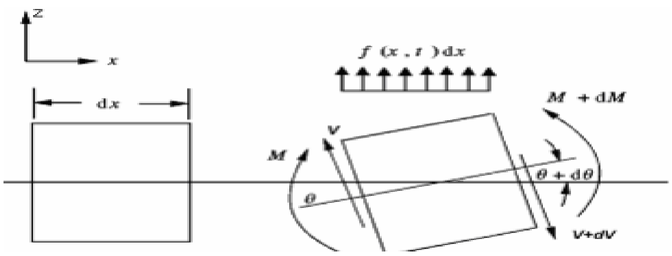


Figure.1. Free body diagram of a section of beam under transverse vibration

$$-(V + dV) + f(x,t) + V = \rho A(x) \frac{d^2w}{dx^2}(x,t) \qquad 2.1$$

Where ρ is the mass density and A is the cross-sectional area of the beam. The moment equation about the y axis leads to

$$(M+dM) - (V + dV)dx + f(x,t)dx\left(\frac{dx}{2}\right) - M = 0 \qquad 2.2$$

By writing.

$$dV=\frac{\partial V}{\partial x}\partial x \text{ and } dM=\frac{\partial M}{\partial x}\partial x$$

And neglecting terms involving second powers in dx, the above equations can be written as

$$-\frac{\partial V(x,t)}{\partial x}+f(x,t)=\rho A(x)\frac{\partial^2 w(x,t)}{\partial t^2} \quad 2.3$$

$$\frac{\partial M(x,t)}{\partial x}-V(x,t)=0 \quad 2.4$$

By using the relation $V=\frac{\partial M}{\partial x}$ from above two equations

$$\frac{\partial^2 M(x,t)}{\partial x^2}+f(x,t)=\rho A(x)\frac{\partial^2 w(x,t)}{\partial t^2} \quad 2.5$$

From the elementary theory of bending of beam, the relationship between bending moment and deflection can be expressed as

$$M(x,t)=EI\frac{\partial^2 w(x,t)}{\partial t^2} \quad 2.6$$

Where E is the Young's modulus and $I(x)$ is the moment of inertia of the beam cross section about the axis. Substituting above two equations, we obtain the equation of the motion for the forced transverse vibration of a non-uniform beam:

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w(x,t)}{\partial t^2}\right]+\rho A(x)\frac{\partial^2 w(x,t)}{\partial t^2}=f(x,t) \quad 2.7$$

For a beam with uniform cross-section, the above equation reduces to

$$\left[EI(x)\frac{\partial^2 w(x,t)}{\partial t^2}\right]+\rho A(x)\frac{\partial^2 w(x,t)}{\partial t^2}=f(x,t) \quad 2.8$$

For free vibration, $f(x,t)=0$, and so the equation of motion becomes

$$c^2\frac{d^4 w}{dx^4}(x,t)+\frac{\partial^2 w(x,t)}{\partial t^2}=0 \quad 2.9$$

Where $c=\sqrt{\frac{EI}{\rho A}}$

Initial Conditions: Since the equation of the motion involves a second order derivative w.r.t time and fourth order derivative w.r.t x, two initial equations as well as four boundary conditions are needed for finding a unique solution for $w(x,t)$. Usually, the values of transverse displacement and velocity are specified as $w_0(x)$ and $w_0(x)\ at\ x=0$. w x t = 0, so that the initial conditions become:

$$w(x,t=0)=w_0(x) \quad 2.10$$

$$\frac{\partial w(x,t=0)}{dt}=w_0 \quad 2.11$$

The free vibration solution can be found using the method of decoupling of variables as

$$w(x,t)=W(x)T(t) \quad 2.12$$

Substituting this equation in the final equation of motion and rearranging leads to

$$\frac{c^2}{W(x)}\frac{d^4W(x)}{dx^4}=-\frac{1}{T(t)}\frac{d^2T(t)}{dt^2}=a=\omega^2 \quad 2.13$$

Where $a=\omega^2$, a positive constant Equation is can be written as two equations

$$\frac{d^4W(x)}{dx^4}-\beta^4W(x)=0 \quad 2.14$$

$$\frac{d^2T(t)}{dt^2}+\omega^2T(x)=0 \quad 2.15$$

Where $\beta^4=\frac{\omega^2}{c^2}=\frac{\rho A\omega^2}{EI}$

The solution to time dependent equation can be expressed as

$$T(x)=A\cos\omega t+A\sin\omega t \quad 2.16$$

Where, A and B are constant that can be found from the initial conditions. For the solution of displacement dependent equation we assume

$$W(x)=Ce^{sx} \quad 2.17$$

Where $C\ and\ s$ are constants, and derive the auxiliary equation as

$$s^4-\beta^4=0$$

The roots of this equation are $s_{1,2}=\pm\beta$ $s_{3,4}=\pm i\beta$

Hence the solution of the equation becomes:

$$W(x)=C_1e^{\beta x}+C_2e^{-\beta x}+C_3e^{i\beta x}+C_4e^{-i\beta x} \quad 2.18$$

$$W(x)=C_1\cos\beta x+C_2\sin\beta x+C_3\cosh\beta x+C_4\,sinh\,\beta x \quad 2.19$$

or

$$W(x)=C_1(cos\beta x+cosh\beta x)+C_2(cos\beta x-cosh\beta x)+C_3(sin\beta x+sinh\beta x)+C_4(sin\beta x-sinh\beta x) \quad 2.20$$

The constants $C_1\ C_2\ C_3\ and\ C_4$ can be found from boundary conditions. The natural frequencies of the beam are computed from:

$$\omega = \beta^2 \sqrt{\frac{EI}{\rho A}} = (\beta l)^2 \sqrt{\frac{EI}{\rho A l^4}} \qquad 2.21$$

Table.1. Value of roots

Roots (i)	$\beta_i l$
1.	1.875104
2.	4.69409
3.	7.85475

The function $W(x)$ is called normal mode or characteristic function of beam and ω is natural frequency of vibration.

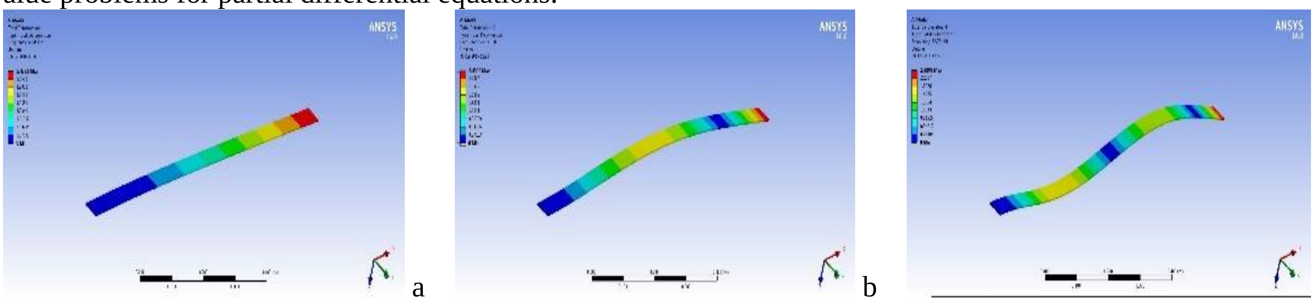
Fixed free beam: The dimensions and the material properties for a uniform fixed free beam (cantilever beam) studied in this paper are: Material of beam = Al, Total length (L) = 1 m, width (B) = 0.05m, height (H) = 0.005 m, moment of inertia (I) = 5.208*10^-10 m^4, Young's Modulus (E) = 70e9, mass per unit length m = 0.6750 kg, mass density = 2700 kg/ m^3.

Putting all required data in Eq 2.21 we get the five frequencies for five modes as shown in table.2.

Table.2. Mode shape frequency

Mode	Frequency in Hz
1	4.1126
2	25.7732
3	72.1659
4	141.4162

Numerical Approach for Transverse Vibration of Fixed Free Beam Using ANSYS: We have investigated the free vibration of fixed free beam using the ANSYS program, a comprehensive finite element package. We used the ANSYS structural package to analyse the vibration of fixed free beam. The finite element method (FEM) is a numerical method for solving problems of engineering and mathematical physics. It is also referred to as finite element analysis (FEA). Typical problem areas of interest include structural analysis, heat transfer, fluid flow, and electromagnetic potential. The analytical solution of these problems generally require the solution to boundary value problems for partial differential equations.



a b c

Figure.2. First 3 Natural Frequencies for different modes of vibration for cantilever beam

Table.3. Mode shape frequency (Ansys)

Mode	Freequency (Hz)
1	4.146
2	25.981
3	41.112

Numerical Approach for Vibration of Fixed Free Beam Using Matlab: We have investigated the free vibration of fixed free beam with and without vibration absorber using the Matlab program. The amplitude response obtained for cantilever beam without absorber has been suppressed using vibration absorber. The absorber has been modelled as spring mass element. The response of a beam subjected to various loads like harmonic and random excitation is controlled using vibration absorber placed at optimum locations along the length of the beam. The response is controlled by choosing the natural frequency of the absorber close to the bending modes.

Matlab Program for Cantilever Beam without Absorber:

```
%
ek(1,1)= 12;     ek(1,2)= 6*L;            ek(1,3)=-12;      ek(1,4)=6*L;
ek(2,1)=ek(1,2); ek(2,2)=4*L^2;             ek(2,3)=-6*L;     ek(2,4)=2*L^2; // element stiffness matrix
ek(3,1)=ek(1,3); ek(3,2)=ek(2,3);          ek(3,3)=12;      ek(3,4)=-6*L;
ek(4,1)=ek(1,4); ek(4,2)=ek(2,4);          ek(4,3)=ek(3,4); ek(4,4)=4*L^2;
ek=ek*kfactor;                                                  // generalized stiffness matrix
%
em(1,1)=13/35;     em(1,2)=11/210*L;
em(1,3)=9/70;      em(1,4)=-13/420*L;
```

```
em(2,1)=em(1,2); em(2,2)=1/105*L^2;   em(2,3)=13/420*L;              // element mass matrix
em(2,4)=-1/140*L^2;
em(3,1)=em( 1,3); em(3,2)=em(2,3); em(3,3)=13/35;
em(3,4)=-11/210*L;
em(4,1)=em(1,4); em(4,2)=em(2,4); em(4,3)=em(3,4); em(4,4)=1/105*L^2;
em=em*m;                                                              // generalized mass matrix
%
```

Matlab Program for Cantilever Beam with Absorber:

```
%
ek(1,1)= 12;     ek(1,2)= 6*L;            ek(1,3)=-12;      ek(1,4)=6*L;
ek(2,1)=ek(1,2); ek(2,2)=4*L^2;           ek(2,3)=-6*L;     ek(2,4)=2*L^2;
ek(3,1)=ek(1,3); ek(3,2)=ek(2,3);         ek(3,3)=12;       ek(3,4)=-6*L;
ek(4,1)=ek(1,4); ek(4,2)=ek(2,4);         ek(4,3)=ek(3,4); ek(4,4)=4*L^2;
ek=ek*kfactor;
%
em(1,1)=13/35;     em(1,2)=11/210*L;
em(1,3)=9/70;      em(1,4)=-13/420*L;
em(2,1)=em(1,2); em(2,2)=1/105*L^2;   em(2,3)=13/420*L;
em(2,4)=-1/140*L^2;
em(3,1)=em( 1,3); em(3,2)=em(2,3); em(3,3)=13/35;
em(3,4)=-11/210*L;
em(4,1)=em(1,4); em(4,2)=em(2,4); em(4,3)=em(3,4); em(4,4)=1/105*L^2;
em=em*m;
% for cantilever dof 1 and dof 2 are fixed
Ka=6.7861;Ma=0.01;
GK(adof,adof)=GK(adof,adof)+ Ka;
GK(adof,ndof)=GK(adof,ndof)-Ka;
GK(ndof,adof)=GK(ndof,adof)-Ka;
GK(ndof,ndof)=GK(ndof,ndof)+Ka;
GM(ndof,ndof)=GM(ndof,ndof)+Ma;
```

2. RESULTS AND DISCUSSION

Numerical (FEA) results:

Table.4. Frequency vs amplitude (Matlab)

Mode	Frequency	Response
1	4.146	0.4383
2	25.981	0.01085
3	41.112	0.00138

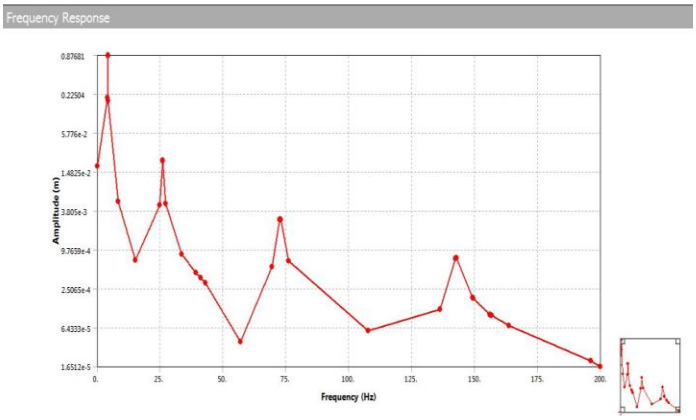


Figure.3. Graphicalrepresentation of the modal frequencies vs amplitude

From the figure.3, the amplitude coresponding to the first three fundamentle modes were obtained using ansys which are listed in Table-4. The amplitude was found to be more at the free end of the beam.

Numerical (MATLAB) results without absorber:

Table.5. Modal frequency

```
Command Window

fn =

    4.1126
   25.7733
   72.1659
```

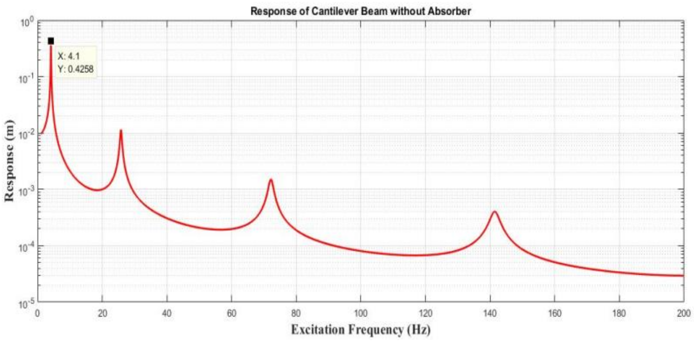


Figure.4. Graphicalrepresentation of the modal frequencies vs amplitude (MATLAB)

From the figure.4, the amplitude coresponding to the first three fundamentle modes were obtained using Matlab which are listed in Table.5. The amplitude was found to be more at the free end of the beam. The same results were verified using ansys.

Numerical (MATLAB) results with absorber:

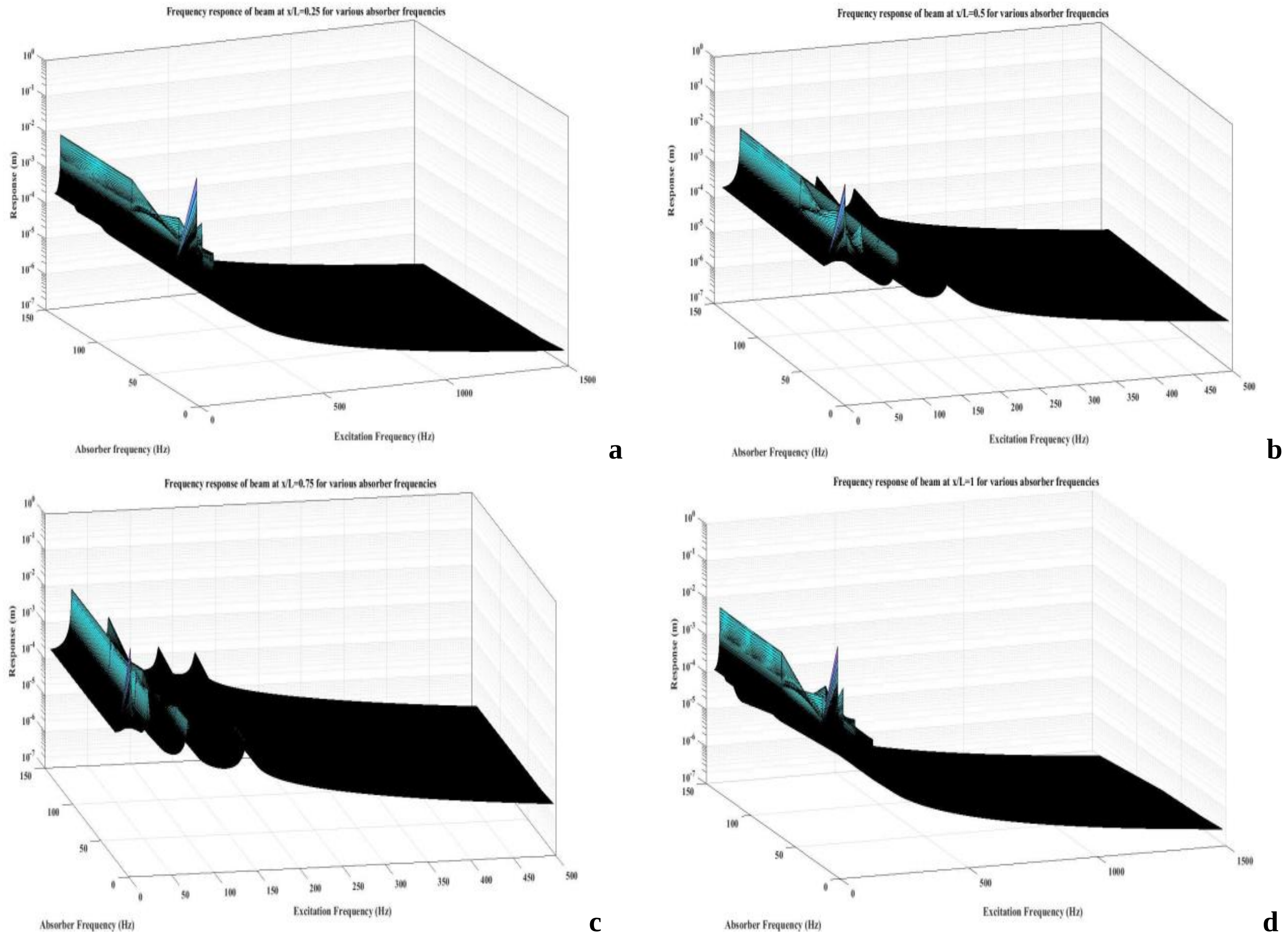


Figure.5. Frequency response of beam for various absorber frequencies (a) X/L=0.25 (b) X/L=0.5 (c) X/L=0.75 (d) X/L= 1

We have tried to suppress the amplitudes by attaching the absorber at various locations of the cantilever beam. The absorber was modelled as a spring mass element. We have tuned the absorber frequency at different locations to supress the fundamental modes. Fig.5(a) shows the reduction in amplitude about 50% for absorber frequency 25.14 Hz. Fig.5(b) shows the reduction in amplitude about 62.14 for absorber frequency 25.14 Hz. Fig.5(c) shows the reduction in amplitude about 63% for absorber frequency 25.14 Hz. Fig.5(d) shows the reduction in amplitude about 64.14% for absorber frequency 25.14 Hz.

Program validation: We have studied the free vibration of fixed free beam by using theoretical approach and the numerical approach using the MATLAB program, it has been found that the relative error between these two approaches are very minute.

The percentage error between the numerical and theoretical methods is shown in Table.6.

Table.6. Percentage Error

Mode	Theoretical frequency in Hz	Numerical frequency from MATLAB program in Hz	Percentage Error %
1	4.1126	4.1126	0
2	25.7721	25.7733	0.0046
3	72.2132	72.1659	0.0065

Since the relative error between the two approaches are very minute, so it can be concluded that theoretical data is in good agreement with numerical results with negligible error.

3. CONCLUSIONS

Initially, we obtained the equation for mode shape frequency theoretically and by analyzing this equation on the fixed free beam which we were used in this paper. The numerical study using the ANSYS and MATLAB program allows investigates the free vibration of fixed free beam to find out mode shape and their frequencies with high accuracy. Therefore it can be concluded that theoretical data is in good agreement with numerical results with negligible error. Moreover, it was found that When the absorber mass was varied, there was no satisfactory reduction in the response at all the nodes. When the absorber frequency is close to the particular mode, then that mode is not excited. The response was found to be less at the centre and at the free end of the beam. The total reduction in response was found to be 64.16% at the free end of the cantilever beam.

REFERENCES

Mohammadi S, Hassanirad A, Applied and Theoretical Cantilever Beam Free Vibration Analysis, World Academy of Science, Engineering and Technology, 61, 2012, 1619-1622.

Samuel S. Stanton, Brian P. Mann, On the dynamic response of beams with multiple geometric and material discontinuities, Duke University, USA, 56, 2010, 889-2256.

Tarsicio Beleandez, Cristian Neipp and Augusto Belea Ndez, Numerical and Experimental Analysis of a Cantilever Beam: a Laboratory Project to Introduce Geometric Nonlinearity in Mechanics of Materials, Int. J. Eng. Ed., 19, 2003, 885 -892.

Thin-Lin Horng, Analytical Solution of Vibration Analysis on Fixed-Free Single-Walled Carbon Nanotube-Based Mass Sensor, Journal of Surface Engineered Materials and Advanced Technology, 2, 2012, 47-52.

Wu J.J and Whittaker A.R, The Natural Frequencies and Mode Shapes of a Uniform Cantilever Beam With Multiple Two-DOF Spring Mass Systems, Journal of Sound and Vibration, 227 (2), 1999, 361-381.

Wu J.S and Chou H.M, Free Vibration Analysis of a Cantilever Beam Carrying Any Number of Elastically Mounted Point Masses With The Analytical-And-Numerical-Combined Method, Journal of Sound and Vibration, 213 (2), 1998, 317-332.

Yoshio Iwata, Toshihiko Komatsuzaki, Satoshi Kitayama, Study on optimal impact damper using collision of vibrators, Kanazawa University, Japan, 336 (5), 2013, 526-986.